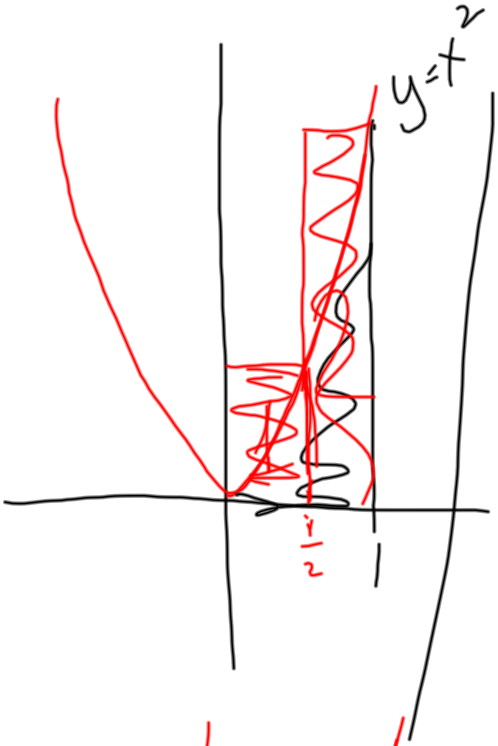
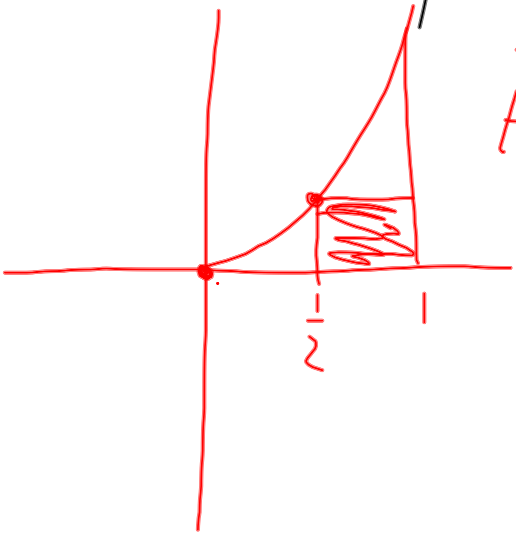


$$A_{R_1} = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$$

$$A_{R_2} = \left(\frac{1}{2}\right)(1) = \frac{1}{2}$$

$$\frac{5}{8}$$

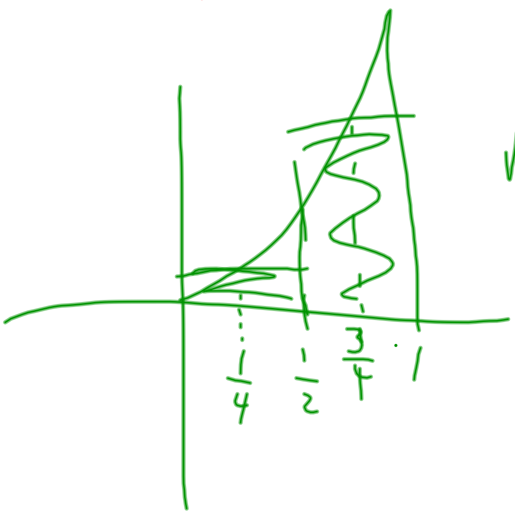


$$A_{R_1} = \left(\frac{1}{2}\right)(0) = 0$$

$$A_{R_2} = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$$

$$\frac{1}{8}$$

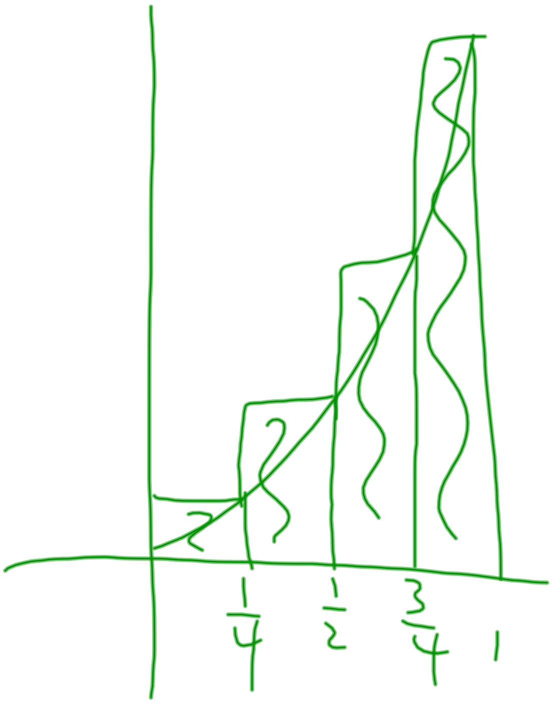
Avg $\frac{3}{8}$



$$A_{R_1} = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = \frac{1}{32}$$

$$A_{R_2} = \left(\frac{1}{2}\right)\left(\frac{9}{16}\right) \quad \frac{9}{32}$$

$$\frac{11}{32} = \frac{5}{16}$$



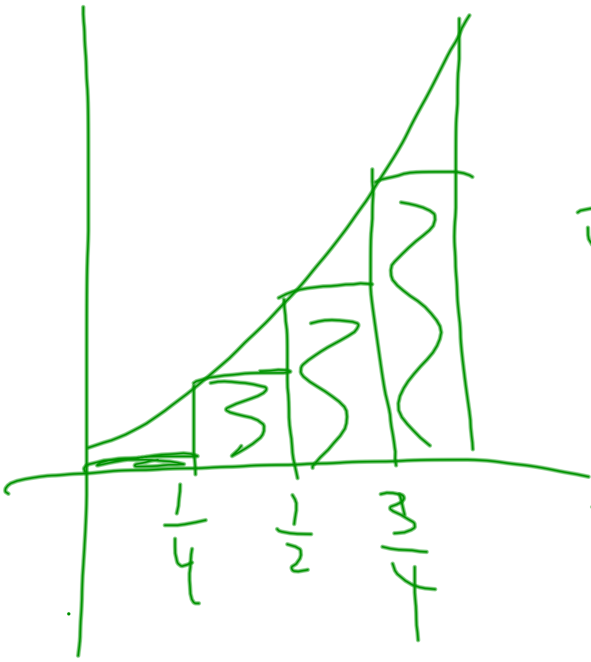
$$\left(\frac{1}{4}\right)\left(\frac{1}{16}\right) = \frac{1}{64} \qquad \frac{1}{64}$$

$$\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16} \qquad \frac{4}{64}$$

$$\frac{1}{4}\cdot\frac{9}{16} = \frac{9}{64} \qquad \frac{9}{64}$$

$$\frac{1}{4}\cdot 1 = \frac{1}{4} \qquad \frac{16}{64}$$

$$\frac{30}{64} = \frac{15}{32}$$



$$\frac{1}{4}\cdot 0 = 0$$

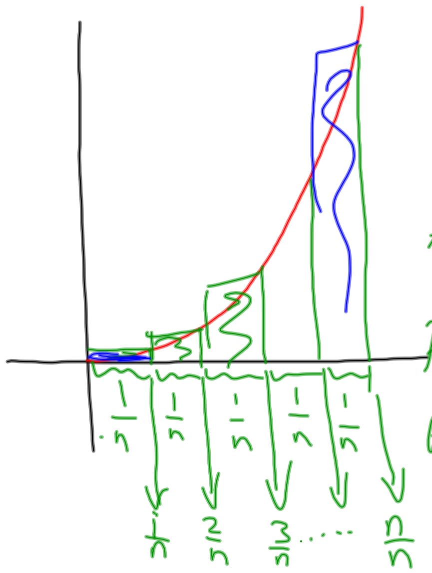
$$\frac{1}{4}\cdot\frac{1}{16} = \frac{1}{64}$$

$$\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$$

$$\frac{1}{4}\cdot\frac{9}{16} = \frac{9}{64}$$

$$\frac{14}{64} = \frac{7}{32}$$

$$\frac{7}{32} \longrightarrow \frac{15}{32}$$



$$A_{R_1} = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right)^2$$

$$A = \left(\frac{1}{n}\right)\left(\frac{2}{n}\right)^2$$

$$A = \left(\frac{1}{n}\right)\left(\frac{3}{n}\right)^2$$

$$\vdots$$

$$A = \frac{1}{n}\left(\frac{n}{n}\right)^2$$

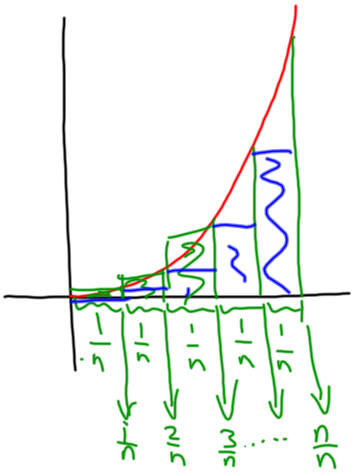
$$A \approx \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right]$$

$$A \approx \frac{1}{n}\left(\frac{1}{n^2}\right)\left[1^2 + 2^2 + 3^2 + \cdots + n^2\right]$$

$$A \approx \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] =$$

$$\text{upper approx} \approx \lim_{n\to\infty} \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] =$$

$$\lim_{n\to\infty} \frac{2n^3+3n^2+n}{6n^3} = \cancel{\frac{2n^3}{6n^3}} = \frac{1}{3}$$



$$A = \text{first area} + \left(\frac{1}{n}\right)(0) - \left(\frac{1}{n}\right)(1)$$

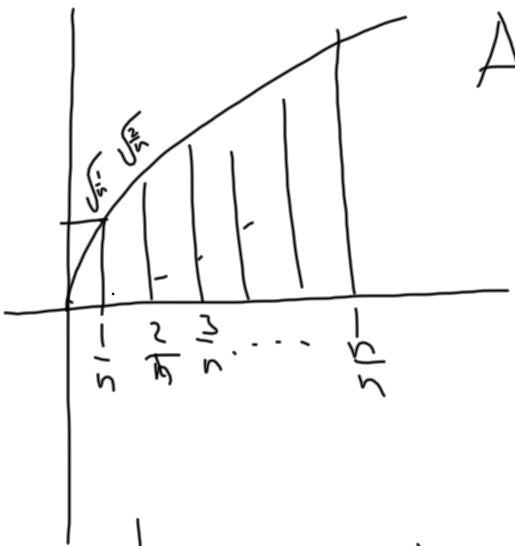
$$A = \frac{2n^3+3n^2+n}{6n^3} - \frac{1}{n}$$

$$\text{lower approx} \le \lim_{n\to\infty}\left(\frac{2n^3+3n^2+n}{6n^3} - \frac{1}{n}\right) = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\text{lower approx } \frac{1}{3} \le A \le \frac{1}{3} \text{ upper approx}$$

6.1/1-8

1) $f(x)=\sqrt{x}$, $[0,1]$



$$A=\left(\frac{1}{n}\right)\left(\sqrt{\frac{1}{n}}\right)+\left(\frac{1}{n}\right)\left(\sqrt{\frac{2}{n}}\right)+\left(\frac{1}{n}\right)\left(\sqrt{\frac{3}{n}}\right)+\dots\left(\frac{1}{n}\right)\left(\sqrt{\frac{n}{n}}\right)$$

$n=2$

$$A=\frac{1}{2}\left(\sqrt{\frac{1}{2}}\right)+\frac{1}{2}\left(\sqrt{\frac{2}{2}}\right)$$

$$=\frac{1}{2}\left(\frac{1}{\sqrt{2}}+1\right)\approx .8535\dots$$

$n=5$

$$A=\frac{1}{5}\left(\sqrt{\frac{1}{5}}\right)+\frac{1}{5}\left(\sqrt{\frac{2}{5}}\right)+\frac{1}{5}\left(\sqrt{\frac{3}{5}}\right)+\frac{1}{5}\left(\sqrt{\frac{4}{5}}\right)+\frac{1}{5}\sqrt{1}$$

$$=\frac{1}{5}\left(\frac{1}{\sqrt{5}}\right)\left(\overset{n=1}{\sqrt{1}}+\overset{n=2}{\sqrt{2}}+\overset{n=3}{\sqrt{3}}+\sqrt{4}+\sqrt{5}\right)$$

$$\sum_{n=1}^{5}\sqrt{n}$$

$$=\frac{1}{5\sqrt{5}}\left(\text{sum}(\text{seq}(\sqrt{n},n,1,5))\right)$$

sum → $\boxed{\text{LIST}}$ → → $\boxed{\text{MATH}}$ 5

seq → $\boxed{\text{LIST}}$ → $\boxed{\text{OPS}}$ 5

5.8/13 $f(x)=\sqrt{x+1}$; $[0,3]$

MVT requires

* continuity on a CLOSED interval
* differentiability on open interval

$f(x)=(x+1)^{1/2}$

$f'(x)=\frac{1}{2}(x+1)^{-\frac{1}{2}}(1)=\frac{1}{2\sqrt{x+1}}$

$f(0)=\sqrt{1}=1$ $(0,1)$

$f(3)=\sqrt{4}=2$ $(3,2)$

$m=\frac{2-1}{3-0}=\frac{1}{3}$

$\frac{1}{2\sqrt{x+1}}=\frac{1}{3} \Rightarrow 3=2\sqrt{x+1}$

$\frac{3}{2}=\sqrt{x+1}$

$\frac{9}{4}=x+1$

$\frac{5}{4}=x$ "my c"

$$A_{R_1} = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$$

$$A_{R_2} = \left(\frac{1}{2}\right)(1) = \frac{1}{2}$$

$$\frac{1}{8} < \text{Area} < \frac{5}{8}$$

$$A_{\text{"}R_1\text{"}} = \left(\frac{1}{2}\right)(0) = 0$$

$$A_{R_2} = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{1}{8}$$

$$\frac{1}{8}$$

→ different shapes
- triangles
- trapezoids

☆ more rectangles

$$A = \left(\frac{1}{4}\right)\left(\frac{1}{16}\right) = \frac{1}{64}$$

$$+ \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{16} = \frac{4}{64}$$

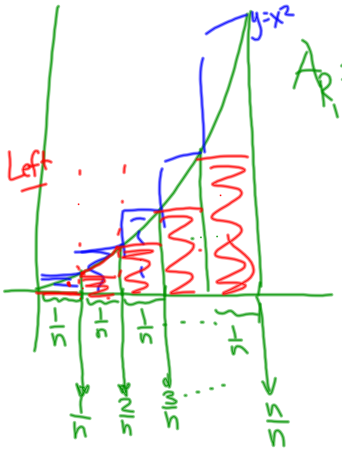
$$+ \left(\frac{1}{4}\right)\left(\frac{9}{16}\right) = \frac{9}{64}$$

$$+ \left(\frac{1}{4}\right)\left(1\right) = \frac{1}{4} = \frac{16}{64}$$

$$\frac{30}{64} = \frac{15}{32}$$

$$\frac{7}{32} < \text{Area} < \frac{15}{32}$$

$$A_{\substack{\text{lower}\\\text{bound}}} = \frac{15}{32} - \left(\frac{1}{4}\right)\left(1\right) = \frac{15}{32} - \frac{8}{32} = \frac{7}{32}$$



$$A_{R_1} = \left(\frac{1}{n}\right)\left(\frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)\left(\frac{2}{n}\right)^2 + \left(\frac{1}{n}\right)\left(\frac{3}{n}\right)^2 + \cdots \left(\frac{1}{n}\right)\left(\frac{n}{n}\right)^2$$

Right

$$A = \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right]$$

$$= \left(\frac{1}{n}\right)\left(\frac{1}{n}\right)^2\left[1^2 + 2^2 + 3^2 + 4^2 + \cdots + (n-1)^2 + (n)^2\right]$$

$$A = \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

n	
2	$\frac{1}{8}\left(\frac{2\cdot3\cdot5}{6}\right) = \frac{5}{8}$
4	$\frac{1}{64}\left[\frac{4\cdot5\cdot9}{6}\right]$ $\frac{1}{64}\left[2\cdot5\cdot3\right] = \frac{30}{64} = \frac{15}{32}$

$$\text{upper bound} \geq \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} =$$

$$\lim_{n\to\infty} \frac{2n^3+3n^2+n}{6n^3} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Area} \leq \frac{1}{3}$$

Lower bound = before Area

$$\frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \left(\frac{1}{n}\right)(0) - \left(\frac{1}{n}\right)(1)$$

$$\text{lower bound} \geq \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} - \frac{1}{n}$$

$$= \frac{1}{3} - \lim_{n\to\infty}\frac{1}{n} = \frac{1}{3}$$

$$\frac{1}{3} \leq \text{Area} \leq \frac{1}{3}$$

6.1 first part + 6.4